

Evaluate  $\int_{-\pi}^{\pi} \tan^2 x \sec^4 x dx$ .

SCORE: \_\_\_\_ / 7 PTS

DIVERGES

$$\int_{-\pi}^{\pi} \tan^2 x \sec^4 x dx = \int_{-\pi}^{-\frac{\pi}{2}} \tan^2 x \sec^4 x dx + \int_{-\frac{\pi}{2}}^0 \tan^2 x \sec^4 x dx + \int_0^{\frac{\pi}{2}} \tan^2 x \sec^4 x dx + \int_{\frac{\pi}{2}}^{\pi} \tan^2 x \sec^4 x dx$$

$$= \lim_{N \rightarrow \frac{\pi}{2}^-} \int_0^N \tan^2 x \sec^4 x dx$$

$u = \tan x \rightarrow x = \tan^{-1} u$   
 $du = \sec^2 x dx$

① EACH

$$= \lim_{N \rightarrow \frac{\pi}{2}^-} \int_0^{\tan N} u^2 (u^2 + 1) du$$

$$= \lim_{N \rightarrow \frac{\pi}{2}^-} \int_0^{\tan N} (u^4 + u^2) du$$

$$= \lim_{N \rightarrow \frac{\pi}{2}^-} \left( \frac{1}{5} u^5 + \frac{1}{3} u^3 \right) \Big|_0^{\tan N}$$

$$= \lim_{N \rightarrow \frac{\pi}{2}^-} \left( \frac{1}{5} \tan^5 N + \frac{1}{3} \tan^3 N \right)$$

$= \infty$

Evaluate  $\int_4^{\infty} \frac{1}{(\sqrt{x}-1)^3} dx = \lim_{N \rightarrow \infty} \int_4^N \frac{1}{(\sqrt{x}-1)^3} dx$

$$\begin{aligned}
 & \begin{aligned}
 & \underline{u = \sqrt{x} - 1} \\
 & x = (u+1)^2 \\
 & dx = 2(u+1)du
 \end{aligned} \\
 & = \lim_{N \rightarrow \infty} \int_1^{\sqrt{N}-1} \frac{2(u+1)}{u^3} du \\
 & = \lim_{N \rightarrow \infty} \int_1^{\sqrt{N}-1} \left( \frac{2}{u^2} + \frac{2}{u^3} \right) du \\
 & = \lim_{N \rightarrow \infty} - \left( \frac{2}{u} + \frac{1}{u^2} \right) \Big|_1^{\sqrt{N}-1} \\
 & = \lim_{N \rightarrow \infty} - \left( \frac{2}{\sqrt{N}-1} + \frac{1}{(\sqrt{N}-1)^2} \right) + 3 \\
 & = 3
 \end{aligned}$$

SCORE: \_\_\_\_ / 7 PTS

① EACH

Evaluate  $\int x \ln(x^2 + 4x + 13) dx = \frac{1}{2}x^2 \ln(x^2 + 4x + 13) - \int \frac{x^2 + 2x^2}{x^2 + 4x + 13} dx$

SCORE: \_\_\_ / 9 PTS

$\frac{u}{\ln(x^2 + 4x + 13)}$	$\frac{dv}{x}$
$\frac{2x+4}{x^2+4x+13}$	$\frac{x^2}{2}$
$\xrightarrow{+}$	
$x^2 + 4x + 13$	$\frac{x-2}{x^3+2x^2}$
	$\frac{x^3+4x^2+13x}{-2x^2-13x}$
	$\frac{-2x^2-8x-26}{-5x+26}$

$$\begin{aligned}
 &= \frac{1}{2}x^2 \ln(x^2 + 4x + 13) - \int \left( x-2 + \frac{-5x+26}{x^2+4x+13} \right) dx \\
 &= \frac{1}{2}x^2 \ln(x^2 + 4x + 13) - \int \left( x-2 + \frac{-5(2x+4)+12(3)}{(x+2)^2+9} \right) dx \\
 &= \frac{1}{2}x^2 \ln(x^2 + 4x + 13) - \frac{1}{2}x^2 + 2x \\
 &\quad + \frac{5}{2} \ln|x^2 + 4x + 13| - 12 \tan^{-1} \frac{x+2}{3} + C
 \end{aligned}$$

① EACH

UNLESS OTHERWISE NOTED

Evaluate  $\int \frac{23-2y}{(y-1)(y^2+y-2)} dy = \int \frac{23-2y}{(y-1)^2(y+2)} dy$

SCORE: \_\_\_\_ / 7 PTS

$$= \int \left( \frac{A}{y-1} + \frac{B}{(y-1)^2} + \frac{C}{y+2} \right) dy$$

$$= \int \left( \frac{-3}{y-1} + \frac{7}{(y-1)^2} + \frac{3}{y+2} \right) dy$$

$$\begin{aligned} 23-2y &= A(y-1)(y+2) \\ &+ B(y+2) \\ &+ C(y-1)^2 \end{aligned}$$

$$y=1: 21=3B \rightarrow B=7$$

$$y=-2: 27=9C \rightarrow C=3$$

$$\text{COEF OF } y^2: 0=A+C$$

$$A=-C=-3$$

SANITY CHECK:

$$y=2$$

$$\text{LHS} = \frac{19}{4}$$

$$\text{RHS} = -3 + 7 + \frac{3}{4} = 4\frac{3}{4} = \frac{19}{4} \checkmark$$

$$= \underbrace{-3 \ln|y-1|}_{\textcircled{1}} - \underbrace{\frac{7}{y-1}}_{\textcircled{2}} + \underbrace{3 \ln|y+2|}_{\textcircled{1}} + C$$

① EACH  
UNLESS OTHERWISE NOTED